 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

THIRD SEMESTER – NOVEMBER 2012

# MT 3810 / 3803 - TOPOLOGY

Date : 01/11/2012 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

**Answer all questions. All questions carry equal marks: 5 x 20 = 100**

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| **01.** | **(a)** | (i) | Let X be a metric space with metric . Show that defined by is also a metric on X.  (OR) (OR) |
|  |  | (ii) | Define a Pseudo metric space on a non-empty set X. Give an example of a pseudo metric which is not a metric. (5) |
|  | **(b)** | (i) | Let X be a complete metric space, and let Y be a subspace of X. Prove that Y is complete iff Y is closed. |
|  |  | (ii) | State and prove Cantor Intersection Theorem. (8+7) |
|  |  |  | (OR) |
|  |  | (iii) | Prove that f is continuous at . |
|  |  | (iv) | Show that f is continuous is open in X whenever G is open in Y. |
| **02.** | **(a)** | (i) | Prove that every second countable space is separable.  (OR) (OR) |
|  |  | (ii) | Define a topology on a non-empty set  with an example. Let  be a topological space and  be an arbitrary subset of . Show that each neighbourhood of intersects . (5) |
|  | **(b)** | (i) | Show that any continuous image of a compact space is compact. |
|  |  | (ii) | Prove that any closed subspace of a compact space is compact. |
|  |  | (iii) | Give an example to show that a compact subspace of a compact space need not be closed. (6+6+3)  (OR)  (OR) |
|  |  | (iv) | Show that a topological space is compact, if every subbasic open cover has a finite subcover. (15) |
| **03.** | **(a)** | (i) | Show that every compact metric space has the Bolzano-Weirstrass property.  (OR) (OR) |
|  |  | (ii) | State and prove Tychanoff's Theorem. (5) |
|  | **(b)** | (i) | Prove that In a sequentially compact metric space every open cover has a Lebesgue number. |
|  |  | (ii) | Show that every sequentially compact metric space is compact. (10+5)  (OR) (OR) |
|  |  | (iii) | State and prove Ascoli’s Theorem (15) |
| **04** | **(a)** | (i) | Show that the product of any non-empty class of Hausdorff spaces is a Hausdorff spaces.  (OR) (OR) |
|  |  | (ii) | Prove that every compact Haurdorff space is normal. (5) (5) |
|  | **(b)** | (i) | Let X be a T1 – space.  Show that X is a normal  each neighbourhood of a closed set F contains the closure of some neighbourhood of F. |
|  |  | (ii) | State and prove Urysohn’s Lemma. (6+9) (6+9)  **(OR)** (OR) |
|  |  | (iii) | If X is a second countable normal space, show that there exists a homeomorphism f of X onto a subspace of . (15) (15) |
| **05.** | **(a)** | (i) | Show that any continuous image of a connected space is connected.  (OR) (OR) |
|  |  |  | Prove that if a subspace of a real line is connected, then it is an internal.(5) |
|  | **(b)** | (i) | Show that the product of any non-empty class of connected spaces is connected. |
|  |  | (ii) | Let X be a Compact Hausdorff Space. Show that X is totally disconnected iff it has open base whose sets are also closed. (6+9) (6+9)  (OR) (OR) |
|  |  | (iii) | State and prove Weierstrass Approximation Theorem. (15) (15) |

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